

RAPID ACCELERATION OF ELECTRONS IN THE MAGNETOSPHERE BY FAST-MODE MHD WAVES

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Abstract. During major magnetic storms enhanced fluxes of relativistic electrons in the inner magnetosphere have been observed to correlate with ULF waves. The enhancements can take place over a period of several hours. In order to account for such a rapid generation of relativistic electrons, we examine the mechanism of transit-time acceleration of electrons by low-frequency fast-mode MHD waves, here the assumed form of ULF waves. Transit-time damping refers to the resonant interaction of electrons with the compressive magnetic field component of the fast-mode waves via the zero cyclotron harmonic. In terms of quasi-linear theory, a kinetic equation for the electron distribution function is formulated incorporating a momentum diffusion coefficient representing transit-time resonant interaction between electrons and a continuous broad band spectrum of oblique fast-mode waves. Pitch angle scattering is assumed to be sufficiently rapid to maintain an isotropic electron distribution function. It is further assumed that there is a substorm-produced population of electrons with energies of the order of 100 keV. Calculations of the acceleration timescales in the model show that fast-mode waves in the Pc4 to Pc5 frequency range, with typically observed wave amplitudes ($\Delta B = 10\text{--}20$ nT), can accelerate the seed electrons to energies of order MeV in a period of a few hours. It is therefore concluded that the mechanism examined in this paper, namely, transit-time acceleration of electrons by fast-mode MHD waves, may account for the rapid enhancements in relativistic electron fluxes in the inner magnetosphere that are associated with major storms.

1. Introduction

There is much current interest in the rapid enhancements of relativistic ($> \text{MeV}$) electrons in the Earth's inner magnetosphere ($3 \leq L \leq 6$) taking place over tens of minutes or a few hours during major magnetic storms [e.g., *Baker et al.*, 1998a; *Rostoker et al.*, 1998; *Liu et al.*, 1999; *Hudson et al.*, 1999,2000]. Part of this interest is due to the fact that relativistic electrons appearing near geostationary orbit ($L = 6.6$) constitute a potential hazard to operational spacecraft [e.g., *Baker et al.*, 1997]. These relativistic electrons are sometimes colloquially referred to as “killer electrons.” Rapid energetic electron enhancements have been observed to correlate closely with ULF waves in the Pc4 (7–22 mHz) or Pc5 (2–7 mHz) frequency ranges. It is therefore reasonable to examine the possible role that ULF waves may have in generating the relativistic electron flux enhancements. *Liu et al.* [1999] have formulated an acceleration mechanism comprising magnetic pumping by ULF waves, while *Hudson et al.* [1999,2000] have proposed a drift-resonant acceleration mechanism involving enhanced ULF waves, modeled by a three dimensional global MHD simulation of the January 10-11, 1997, coronal-mass-ejection-driven magnetic cloud event. Notwithstanding these studies, the acceleration mechanism of relativistic electrons in the inner magnetosphere is not yet fully understood. It is the purpose of the present paper to examine the role of ULF waves in accelerating electrons in the magnetosphere from a new standpoint. Here we examine the “transit-time acceleration” of electrons by low-frequency, oblique, fast-mode (magnetosonic) MHD waves. Transit-time acceleration in association with

“transit-time damping” has been studied, for instance, by *Stix* [1962], *Fisk* [1976], and *Achterberg* [1981]. The basic physical mechanism of transit-time damping, which is a resonant form of Fermi acceleration and can be regarded as the magnetic analogue of Landau damping, is discussed in detail by *Miller* [1997]; the name transit-time damping arises because the gyroresonance condition defining the process can be expressed as $\lambda_{\parallel}/v_{\parallel} \approx T$, where v_{\parallel} is the parallel component of particle velocity, λ_{\parallel} is the parallel wavelength, and T is the wave period. Thus the wave-particle interaction is strongest when the particle transit-time across the wave compression is approximately equal to the period. It is the compressive magnetic field component of the fast-mode wave that allows for the effect of transit-time damping [*Fisk*, 1976; *Achterberg*, 1981; *Miller et al.*, 1996]. While transit-time damping has been utilized as a mechanism for accelerating energetic particles in the interplanetary medium [*Fisk*, 1976], for accelerating electrons in solar flares [*Miller et al.*, 1996; *Miller*, 1997], and for accelerating cosmic ray particles [*Schlickeiser and Miller*, 1998], it has not been examined as a possible acceleration mechanism of electrons in the magnetosphere. It will be shown in this paper, in fact, that transit-time damping of fast-mode MHD waves (here the assumed form of ULF waves) is a viable mechanism for generating the aforementioned rapid enhancements of relativistic electrons in the inner magnetosphere. It will be assumed that electrons of energies ~ 100 keV that are injected near geosynchronous orbit as a result of substorm activity [e.g., *Baker et al.*, 1989, 1998b] form the source population for the relativistic ($> \text{MeV}$) electrons that are subsequently observed.

The structure of ULF waves in the Earth’s magnetosphere is complex. Broadly,

MHD waves in the dipole magnetosphere are characterized by “toroidal” and “poloidal” modes, and, in general, these modes are coupled. Toroidal modes relate to transverse ULF waves propagating along field lines, while poloidal modes relate to global compressional waves associated with radial oscillations of the field lines. Field line resonance (FLR) theory describes the toroidal pulsations as transverse Alfvén waves standing on dipole flux tubes with fixed ends in the ionosphere. Extensive theory has accumulated pertaining to field line resonances, global compressional modes, and associated wave excitation mechanisms [e.g., see *Kivelson and Southwood*, 1986, *Krauss-Varban and Patel*, 1988, *Lee and Lysak*, 1989, and references therein]. *Anderson et al.* [1990] give a historical review of observations of ULF waves in the magnetosphere and also present results of a statistical study of Pc3–5 pulsations during the period from August 24, 1984, to December 7, 1985. It should be emphasized here that unlike ULF waves observed during relatively quiet magnetic conditions, storm-associated ULF waves characteristically have large compressional components. It is these components that engender the transit-time acceleration mechanism presented in this paper.

With respect to the typical motion of energetic electrons in the inner magnetosphere, these electrons gyrate around their magnetic field line and bounce back and forth along their field line between mirror points, while executing an eastward drift about the Earth. Consequently, since in the interests of rendering the theory presented herein tractable we shall assume a constant background magnetic field, our formulation applies to compressional MHD waves interacting with electrons that mirror relatively close to the equator.

We are concerned with the fast-mode MHD branch in plasma wave theory [e.g., *Swanson*, 1989] and, in particular, with fast-mode waves on the $\omega \ll \Omega_i$ section of the branch, where Ω_i is the proton gyrofrequency. Such waves have the dispersion relation,

$$\omega = kv_A, \quad (1)$$

where ω , k , and v_A are the wave frequency, the wave number, and the Alfvén speed, respectively. In general, the condition for gyroresonant interaction between electrons and a wave of frequency ω is

$$\omega - k_{\parallel}v_{\parallel} = n|\Omega_e|/\gamma, \quad (2)$$

where v_{\parallel} is the electron parallel velocity component, $k_{\parallel} = k \cos \theta$ is the parallel wave number, θ is the wave propagation angle, $|\Omega_e|$ is the electron gyrofrequency, γ is the Lorentz factor, and $n (= 0, \pm 1, \pm 2, \dots)$ denotes the cyclotron harmonic. The compressive component of the wave magnetic field can interact with electrons through the $n = 0$ resonance [e.g., *Miller et al.*, 1996], in which case (2) reduces to

$$\omega = k_{\parallel}v_{\parallel}. \quad (3)$$

Equation (3) is the gyroresonance condition that defines transit-time damping [*Stix*, 1962; *Fisk*, 1976; *Achterberg*, 1981]. From (1) and (3) it follows that

$$v_{\parallel} = v_A / \cos \theta, \quad (4)$$

from which follows the important necessary threshold condition for resonance,

$$v > v_A, \quad (5)$$

where v is the particle speed. Condition (5) states that for electrons with any pitch-angle interacting with fast-mode MHD waves propagating at any angle θ , resonance is only possible for electrons with speeds exceeding the Alfvén speed. The equivalent minimum-energy condition can be conveniently written,

$$E > E_{\min},$$

$$E_{\min} = (1 - \beta_A^2)^{-1/2} - 1 \approx \beta_A^2/2, \quad (6)$$

where E is the electron kinetic energy in units of rest-mass energy and β_A is the Alfvén speed in units of the speed of light. Values for the parameter β_A and the minimum energy E_{\min} that are representative of the inner magnetosphere are given in Table 1. In Table 1, we set $N_0 = 10 \text{ cm}^{-3}$ as the particle number density in the inner magnetosphere outside the plasmasphere, and we use the equatorial (dipole) magnetic field value $B_0 = 3.12 \times 10^{-5}/L^3 \text{ T}$. With regard to the background electron population, in order for the fast-mode waves to accelerate a small fraction of the electrons in the tail of the distribution rather than to produce a bulk heating of the population, it is required that $v_A > v_{th}$, where v_{th} is a characteristic thermal speed. Taking the background electron temperature in the magnetosphere to be $T_e \lesssim 1 \text{ eV}$, we find from Table 1 that the required condition $v_A > v_{th}$ is satisfied. In addition, since we are assuming a substorm-produced source of electrons with energies $\sim 100 \text{ keV}$, Table 1 shows that the minimum-energy condition (6) is well satisfied; that is, the condition $v \gg v_A$ holds.

The analysis of transit-time damping of fast-mode waves that is carried out in this paper and presented in the following section is based on quasi-linear theory; this is

an approximation that requires justification. In numerical simulations, *Miller* [1997] has found that quasi-linear theory provides an accurate description of transit-time acceleration even when the energy density of the fast-mode wave turbulence is almost equal to the ambient magnetic field energy density $((\Delta B/B_0)^2 \lesssim 1)$. Thus, although the analysis presented here is based formally on “small-amplitude” turbulence $((\Delta B/B_0)^2 \ll 1)$, the results are applicable to the large-amplitude ULF waves typically observed during magnetic storms.

2. Electron Momentum Diffusion Equation

Consider energetic charged particles in a uniform magnetic field with superimposed small-amplitude plasma turbulence. By using the quasi-linear approximation [*Kennel and Engelmann*, 1966; *Lerche*, 1968], the pitch angle averaged particle distribution function $F(p, t)$ can be shown to satisfy the kinetic (Fokker-Planck) equation

$$\frac{\partial F}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D(p) \frac{\partial F}{\partial p} \right), \quad (7)$$

where

$$D(p) = \frac{1}{2} \int_{-1}^1 D_{pp} d\mu. \quad (8)$$

In (7) and (8), p is the relativistic momentum of the particle in units of rest-mass momentum given by $p = \gamma v/c$, where v is the particle speed and $\gamma = (1 - v^2/c^2)^{-1/2} = (1 + p^2)^{1/2}$ is the Lorentz factor, with c being the speed of light; t is time; μ is the cosine of the pitch angle; and D_{pp} is the momentum diffusion coefficient, which depends on the properties of the wave turbulence. In the derivation of (7), it has been assumed that

the rate of pitch angle scattering is large enough to isotropize the distribution function, and the pitch angle has been eliminated from the equation by averaging with respect to μ . The distribution function F is normalized so that $4\pi p^2 F(p, t) dp$ is the number of the particles per unit volume in the momentum interval dp . It has also been assumed in deriving (7) that there are no energy losses, that no particles escape from the system, and that there are no additional particle sources or sinks.

Associated with the (averaged) momentum diffusion coefficient $D(p)$ in (7) and (8) is the acceleration timescale,

$$T_A = p^2 / D(p). \quad (9)$$

In this paper we consider two forms for the transit-time damping diffusion coefficient, given by *Miller et al.* [1996] and *Schlickeiser and Miller* [1998], respectively. Assuming a continuous spectrum of oblique, low-frequency ($\omega \ll \Omega_i$), fast-mode waves and assuming isotropic turbulence and integrating over wave propagation angle, *Miller et al.* [1996] obtain a diffusion coefficient D_{pp} for transit-time damping of fast-mode waves by electrons that can be expressed in the form

$$D_{pp} = \frac{\pi}{16} \Omega_i R \left(\frac{c\langle k \rangle}{\Omega_i} \right) \gamma^2 \beta \beta_A^2 \left(1 - \frac{\beta_A^2}{\beta^2 \mu^2} \right) \frac{(1 - \mu^2)^2}{|\mu|}, \quad (10)$$

where $\Omega_i = eB_0/(m_i c)$ is the proton gyrofrequency, with B_0 being the ambient magnetic field strength, m_i being the proton rest mass, and e being the electronic charge; $R = (\Delta B/B_0)^2$ is the ratio of the turbulent wave energy to magnetic field energy, with ΔB being the average fast-mode wave amplitude; $c\langle k \rangle/\Omega_i$ is the mean dimensionless wave number of the wave spectrum; $\beta = v/c$; and $\beta_A = v_A/c$ where

$v_A = B_0/(4\pi N_0 m_i)^{1/2}$ is the Alfvén speed, with N_0 being the particle number density.

Substituting (10) into (8) and setting

$$x = \beta_A/\beta \quad (11)$$

yields the result

$$D(p) = \frac{\pi}{16} \Omega_i R \left(\frac{c\langle k \rangle}{\Omega_i} \right) \gamma^2 \beta \beta_A^2 g(x) , \quad (12)$$

where

$$g(x) = (1 + 2x^2) \log_e \left(\frac{1}{x} \right) + x^2 + \frac{x^4}{4} - \frac{5}{4} , \quad (13)$$

for $x < 1$. The function $g(x)$ can be regarded as an efficiency factor [Miller *et al.*, 1996], which relates to the velocity-dependent fraction of electrons that can resonate with fast-mode waves having the assumed spectrum; $g(x) = 0$ for $x \geq 1$, and in the limit as $\beta \rightarrow 1$, $g(x) \rightarrow \log_e(1/\beta_A) - 5/4$, approximately, since $\beta_A \ll 1$. Therefore, for values of β_A appropriate to the Earth's magnetosphere (see Table 1) for $3 \leq L \leq 6.6$, with $N_0 = 10 \text{ cm}^{-3}$, the function $g(x)$ approaches values in the range from 2.4 to 4.7 for highly relativistic electrons.

Setting $\langle k \rangle = \langle \omega \rangle / v_A$ in (12), where $\langle \omega \rangle$ is the mean angular frequency (rad/s), from (9) and (12) we find that the acceleration timescale T_A can be written as

$$T_A = \frac{8}{\pi^2} \frac{1}{\langle f_w \rangle} \frac{1}{R} \frac{1}{x g(x)} , \quad (14)$$

where $\langle f_w \rangle = \langle \omega \rangle / 2\pi$ is the mean wave frequency (in millihertz). Later in this section and in the numerical results presented below, we shall find it convenient to use the previously introduced dimensionless kinetic energy $E = E_k / (m_e c^2) = \gamma - 1$, where E_k

is the electron kinetic energy and m_e is the electron rest mass; we shall require the relation,

$$\beta = [E(E + 2)]^{1/2} / (E + 1). \quad (15)$$

Schlickeiser and Miller [1998] assume that the fast-mode wave turbulence is isotropic and Kolmogorov-like, with a power law spectral energy density distribution in wave number k . Specifically, the spectral energy density W is assumed to take the form

$$W(k) \propto k^{-q}, \quad k > k_{min}, \quad (16)$$

where $q(> 1)$ is the spectral index and k_{min} is some minimum wave number.

Corresponding to the Kolmogorov-like spectrum (16), the momentum diffusion coefficient D_{pp} , as given by *Schlickeiser and Miller* [1998], can be written

$$D_{pp} = \frac{\pi}{4}(q-1)\Omega_i R \left(\frac{ck_{min}}{\Omega_i} \right)^{q-1} \left(\frac{m_e}{m_i} \right)^{q-2} \gamma(\gamma\beta)^{q-1} \beta_A^2 h(\mu, x), \quad (17)$$

where

$$h(\mu, x) = H(|\mu| - x) \frac{1 - \mu^2}{|\mu|} \left[1 + \frac{x^2}{\mu^2} \right] \left[(1 - \mu^2) \left(1 - \frac{x^2}{\mu^2} \right) \right]^{q/2} \int_{\lambda}^{\infty} \frac{J_1^2(s)}{s^{1+q}} ds, \quad (18)$$

with

$$\lambda = \left(\frac{ck_{min}}{\Omega_i} \right) \left(\frac{m_e}{m_i} \right) \gamma\beta(1 - \mu^2)^{1/2} \left(1 - \frac{x^2}{\mu^2} \right)^{1/2}. \quad (19)$$

In (17)–(19), ck_{min}/Ω_i is the minimum dimensionless wave number of the wave spectrum, H is the Heaviside unit function, and J_1 is the Bessel function of the first kind of order unity. Substitution of (17) into (8) yields

$$D(p) = \frac{\pi}{4}(q-1)\Omega_i R \left(\frac{ck_{min}}{\Omega_i} \right)^{q-1} \left(\frac{m_e}{m_i} \right)^{q-2} \gamma(\gamma\beta)^{q-1} \beta_A^2 I(x, \beta_A, k_{min}), \quad (20)$$

where

$$I(x, \beta_A, k_{min}) = \begin{cases} c_1(q) \log_e \left(\frac{1}{x} \right), & 1 < q \leq 2 \\ c_2(q) (\gamma/\beta)^{2-q} \left(\frac{ck_{min}}{\Omega_i} \right)^{2-q} \left(\frac{m_e}{m_i} \right)^{2-q} \log_e \left(\frac{1}{x} \right), & q > 2 \end{cases} \quad (21)$$

with

$$\begin{aligned} c_1(q) &= 2^{1-q} \frac{q}{4-q^2} \frac{\Gamma(q)\Gamma(2-q/2)}{\Gamma^3(1+q/2)}, & 1 < q < 2 \\ c_1(2) &= 3/4, \\ c_2(q) &= \frac{2q^2 - 3q + 4}{4q(2q-3)}, & q > 2 \end{aligned} \quad (22)$$

where Γ is the gamma function.

In (20) we set the minimum dimensionless wave number $k_{min} = 2\pi f_{min}/v_A$, where f_{min} is the minimum wave frequency (in millihertz). From (9) and (20) the acceleration timescale for transit-time damping associated with the wave spectrum (16) is found to be

$$T_A = \frac{8}{q-1} \frac{1}{(2\pi)^q} \frac{1}{\Omega_i} \frac{1}{R} \left(\frac{\Omega_i}{f_{min}} \right)^{q-1} \left(\frac{m_i}{m_e} \right)^{q-2} \frac{1}{\gamma^{q-2}} \frac{1}{x^{3-q} I}, \quad (23)$$

where I is given by (21) with ck_{min}/Ω_i replaced by $(f_{min}/\Omega_i)(2\pi/\beta_A)$.

It should be noted that while the transit-time damping diffusion coefficients (12) and (20) may appear different, they are, in fact, approximately equivalent. Since the coefficient (12) employs an average wave frequency, while coefficient (20) employs a minimum frequency and a Kolmogorov spectral index, it is convenient to utilize both coefficients in order to retain some flexibility in constructing the acceleration timescale profiles and comparing the results with observations.

Finally, it is useful to relate the mean energy change of a particle $\langle \dot{E} \rangle$ to the

acceleration timescale T_A . Associated with the momentum diffusion process given by (7) – (8), the mean energy change [Tsytoich, 1977; Achterberg, 1981] is given by

$$\begin{aligned}\langle \dot{E} \rangle &= \frac{1}{p^2} \frac{\partial}{\partial p} (\beta p^2 D(p)) \\ &\approx \frac{\sigma \beta}{p} D(p),\end{aligned}\tag{24}$$

and, hence, by (9) we derive the result,

$$\langle \dot{E} \rangle \approx \frac{\sigma \beta p}{T_A} = \frac{\sigma E(E+2)}{(E+1)T_A},\tag{25}$$

where σ is a factor such that $\sigma = 4$ corresponding to the diffusion coefficient (12), and $\sigma = 2 + q$ corresponding to (20). The approximation in the second line of (24) follows from the fact that, for the electron energies considered in this paper, the functions g and I vary only slightly with x .

3. Numerical results

The acceleration timescale T_A depends on a number of parameters. Both results (14) and (23) depend on the average wave amplitude ΔB , the electron kinetic energy E , the background plasma number density N_0 , and the location L . In addition, (14) depends on the mean wave frequency $\langle f_w \rangle$, while (23) depends on the the minimum wave frequency f_{min} and the turbulence spectral index q . With regard to typical wave amplitudes of Pc-5 pulsations during major magnetic storms, *Barfield and McPherron* [1978] and *Engebretson and Cahill* [1981] report $\Delta B \approx 10$ nT, while *Higuchi et al.* [1986] report typical values $\Delta B \approx 70 - 90$ nT corresponding to the maximum power spectral

densities in the frequency range 5 – 12 mHz. *Baker et al.* [1998a] report ULF waves in the frequency range 2 – 20 mHz having amplitudes $\Delta B \approx 50$ nT, rising to $\Delta B \approx 200$ nT at times.

We assume a substorm-produced seed electron population with energies in the range 100 – 300 keV, which corresponds to the dimensionless kinetic energy E in the approximate range $0.2 < E < 0.6$. From result (25), it follows that electrons with energies in such a range accelerate to energies in the range from 1 MeV to 2 MeV, approximately, over the timescale $T_A(E)$ where $0.2 < E < 0.6$. In Figure 1, $N_0 = 10 \text{ cm}^{-3}$, and for the specified mean wave frequency $\langle f_w \rangle = 10$ mHz, curves are plotted showing T_A , given by (14), as a function of energy E (eV), at the locations $L = 3, 4, 5, 6.6$, for each of the wave amplitudes $\Delta B = 10, 20, 50$ nT; for reference, a (dashed) line is shown corresponding to a time of one day. In general, for a fixed energy E , the timescale T_A is seen to increase as the value of L decreases, and, as expected, T_A decreases as the wave amplitude ΔB increases. Figure 1 indicates, in particular, that at $L = 6.6$, for the parameter values $N_0 = 10 \text{ cm}^{-3}$ and $\langle f_w \rangle = 10$ mHz, the timescales for accelerating seed electrons of energies ~ 100 keV to energies ~ 1 MeV are approximately 6 days, 1.5 days, and 5.8 hours corresponding to the respective wave amplitudes $\Delta B = 10, 20, 50$ nT. The aforementioned respective times assume the approximate values 2 days, 12 hours, and 2 hours if the value of $N_0 = 1 \text{ cm}^{-3}$ is specified for the background plasma number density (N.B. It could be argued that $N_0 = 10 \text{ cm}^{-3}$ is too high a generic value for the background plasma number density, and that $N_0 = 1 \text{ cm}^{-3}$ is a more representative value).

In Figure 2, we show the variation of T_A , as given by (14), as a function of the wave amplitude ΔB (nT), for a fixed value of particle energy, $E = 1/4$ (or $\beta = 0.6$). The upper, middle, and lower panels of Figure 2 correspond respectively to the mean wave frequencies $\langle f_w \rangle = 2, 10$, and 22 mHz. The decrease in acceleration timescale with increase in mean frequency $\langle f_w \rangle$, as indicated by formula (14), is clearly shown in Figure 2. Figure 2 can be used as an illustration of the wave amplitudes ΔB , at a given location L , and for a given mean wave frequency $\langle f_w \rangle$, that correspond to a particular timescale T_A for the generation of electrons of energies $\gtrsim 1$ MeV from seed electrons of energies $\gtrsim 100$ keV. In Table 1, corresponding to (14), the required wave amplitudes ΔB are given that correspond to a timescale $T_0 = 10$ hours for this generation process, corresponding to the mean wave frequencies $\langle f_w \rangle = 2, 10$, and 22 mHz at the specified locations, and with $N_0 = 10 \text{ cm}^{-3}$. In particular, we note that at $L = 6.6$, corresponding to the respective mean wave frequencies $\langle f_w \rangle = 10, 22$ mHz, the required wave amplitudes are $\Delta B = 39, 26$ nT; corresponding to $N_0 = 1 \text{ cm}^{-3}$, these respective wave amplitudes are $\Delta B = 22, 15$ nT.

In Figure 3, for $N_0 = 10 \text{ cm}^{-3}$, the acceleration timescale T_A (sec) given by (23) is plotted as a function of particle energy E , for the minimum wave frequencies $f_{min} = 2, 10$ mHz, for the spectral indices $q = 3/2, 5/3, 5/2, 4$, at each of the locations $L = 3, 6.6$, and for the mean wave amplitude $\Delta B = 20$ nT. As can be observed from Figure 3, the timescale T_A decreases as both f_{min} and L increase; lower values of T_A are also generally favoured by lower q -values. The curves in Figure 3 show that, corresponding to $\Delta B = 20$ nT, the timescale T_A at $L = 6.6$ is of the order of a few hours,

for values of q in the range $3/2 < q < 5/3$. In Table 2, values of the wave amplitudes ΔB (nT) are given that correspond to the value of the acceleration timescale T_A given by (23) equal to 10 hours, for the specified values of f_{min} , q , and L , with $E = 1/4$ (or $\beta = 0.6$), and $N_0 = 10 \text{ cm}^{-3}$. Thus, Table 2, which corresponds to (23), effectively gives the required wave amplitudes ΔB to generate relativistic ($\gtrsim 1 \text{ MeV}$) electrons from seed ($\gtrsim 100 \text{ keV}$) electrons in a timescale of 10 hours, for the specified values of the remaining parameters. For instance, at $L = 6.6$, for a minimum wave frequency $f_{min} = 10 \text{ mHz}$, and with q in the range $3/2 < q < 5/3$, the required wave amplitudes are in the range $5.4 \text{ nT} < \Delta B < 7.1 \text{ nT}$.

4. Discussion

The present paper is a new examination of ULF waves as a possible rapid acceleration mechanism of electrons in the inner magnetosphere during storms. Specifically, we take the assumed form of ULF waves to be fast-mode (magnetosonic) MHD waves, and analyze the mechanism of transit-time acceleration of electrons under magnetic storm conditions. We assume that the seed electrons in the process have energies in the range $100 - 300 \text{ keV}$, and are produced by substorm activity. In accordance with quasi-linear theory and a test particle approach, a simple model kinetic equation (7) is formulated in which momentum diffusion is due to the gyroresonant transit-time interaction between electrons and fast-mode MHD turbulence. A continuous broad-band spectrum of oblique fast-mode waves is assumed, and it is further supposed that pitch-angle scattering is sufficiently rapid to maintain an isotropic particle

distribution function. The model calculations applied to the inner magnetosphere show that the mechanism under consideration, namely transit-time damping of fast-mode MHD waves, can accelerate source electrons with energies 100 – 300 keV to relativistic electrons with energies exceeding 1 MeV, in a timescale of a few hours if the wave amplitudes are of the order of $\Delta B = 10 - 20$ nT. Since observed amplitudes of ULF waves during storm-time are in this range, it is concluded that transit-time damping of fast-mode MHD waves, as the agent of ULF wave activity, could play an important role in generating the observed increases of relativistic electrons during major storms.

We note that the models formulated by *Liu et al.* [1999] and *Hudson et al.* [1999a, b] also show that ULF waves could be instrumental in energizing relativistic electrons under storm conditions, though their approaches are quite different from that adopted here. *Liu et al.* [1999] formulate an acceleration mechanism comprising magnetic pumping with global ULF waves as the energy source and pitch-angle scattering as the catalyst, while *Hudson et al.* [1999a, b] propose a mechanism, further investigated by *Elkington et al.* [1999], in which electrons are adiabatically accelerated through a drift-resonance via interaction with toroidal-mode ULF waves.

We caution that the calculations in the present paper are based on an approximate, timescale analysis. A more complete investigation of electron acceleration by transit-time damping of fast-mode waves entails the full solution of a kinetic equation of the form (7), appropriately modified by the inclusion of terms representing particle and energy losses under storm conditions.

Aside from the aforementioned ULF wave mechanisms, other energization

mechanisms have been previously proposed to account for relativistic electron enhancements during storms, e.g., see *Li et al.* [1997] and *Summers and Ma* [1999] for brief summaries. Moreover, various types of storm-related energetic electron events have been observed [e.g., *Baker et al.*, 1997; 1998b; *Reeves*, 1998; *Reeves et al.*, 1998]. The rapid acceleration mechanism presented in this paper appears well suited to major storms that produce coherent global oscillations in the magnetosphere in the Pc-4 to Pc-5 frequency range. In contrast, the gradual acceleration process occurring over a few days involving gyroresonant electron-whistler-mode chorus interaction [*Summers et al.*, 1998, 1999; *Summers and Ma*, 1999; see also *Ma and Summers*, 1998] is expected to apply to moderate storms having long-lasting recovery phases.

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Figure 1. Acceleration timescale T_A (sec) as given by (14), as a function of the electron kinetic energy E (eV), at the indicated locations L , for the average wave amplitudes $\Delta B = 10, 20, 50$ nT. The mean wave frequency $\langle f_w \rangle = 10$ mHz.

Figure 2. Acceleration timescale T_A (sec) as given by (14), as a function of the average wave amplitude ΔB (nT), at the indicated locations L , for the mean wave frequencies $\langle f_w \rangle = 2, 10, 22$ mHz. The parameter $\beta = 0.6$.

Figure 3. Acceleration timescale T_A (sec) as given by (23), as a function of the electron kinetic energy E (eV), at the indicated locations L , for the given values of the spectral index q , and for the minimum wave frequencies $f_{min} = 2, 10$ mHz. The average wave amplitude $\Delta B = 20$ nT.

Table 1. The required average wave amplitudes ΔB (nT), as calculated from (14), that correspond to an acceleration timescale T_A of about 10 hours, at the given locations L , and for the mean wave frequencies $\langle f_w \rangle = 2, 10, 22$ mHz. The parameter $\beta = 0.6$. Also given are the ambient magnetic field strength B_0 (10^{-7} T), the dimensionless Alfvén speed $\beta_A = v_A/c$, and the minimum energy E_{min} (eV). The latter value is calculated from $E_{min} \text{ (eV)} = 512 \times 10^3 E_{min}$ where E_{min} is given by (6).

				$\langle f_w \rangle = 2$	$\langle f_w \rangle = 10$	$\langle f_w \rangle = 22$
L	B_0	β_A	$E_{min}(eV)$	ΔB	ΔB	ΔB
3	11.6	2.67×10^{-2}	183	427	191	129
4	4.85	1.13×10^{-2}	33	222	99	67
5	2.50	5.75×10^{-3}	9	147	66	44
6.6	1.10	2.53×10^{-3}	2	87	39	26

Table 2. The required average wave amplitudes ΔB (nT), as calculated from (23), that correspond to an acceleration timescale T_A of about 10 hours, at the given locations L , for the given values of the spectral index q , and the minimum wave frequencies $f_{min} = 2, 10$ mHz. The parameter $\beta = 0.6$.

L	$f_{min} = 2$ mHz				$f_{min} = 10$ mHz			
	$q = 1.5$	$q = 5/3$	$q = 2.5$	$q = 4.0$	$q = 1.5$	$q = 5/3$	$q = 2.5$	$q = 4.0$
3	11	24	201	174	7.2	14	90	78
4	9.5	18	115	100	6.3	11	51	45
5	8.8	15	76	66	5.9	8.8	34	30
6.6	8.1	12	47	40	5.4	7.1	21	18







